



# ANALYSIS SEMINAR

\*\*\*\*\*

**Injectivity of the modules  $L^p(G)$**

by

**H. G. Dales**

**University of Lancaster, UK**

\*\*\*\*\*

## ABSTRACT

Let  $A$  be a Banach algebra, and let  $E$  be a Banach  $A$ -bimodule. A basic question is to determine when  $E$  is injective (in a sense that I shall explain). A fundamental result of Helemski is that  $E$  is injective whenever  $A$  is an amenable Banach algebra and  $E$  is a dual module. The following has been open for a long time:

Suppose that all (or some specific) dual Banach  $A$ -bimodules are injective. Does it always follow that  $A$  is amenable?

Let  $G$  be a locally compact group, and let  $L^1(G)$  be the standard group algebra. Then  $L^p(G)$  is a Banach  $L^1(G)$ -bimodule for each  $p \geq 1$ , and it is a dual module when  $p > 1$ . Suppose that  $G$  is an amenable group. Then  $L^1(G)$  is an amenable Banach algebra, and so  $L^p(G)$  is injective for each  $p > 1$ . We can now show that, conversely,  $G$  is amenable whenever  $L^p(G)$  is injective for some  $p > 1$ . Our approach is to use the theory of ‘multi-norms’, which I shall go over briefly. The talk is based on the following paper:

H. G. Dales, M. Daws, H. L. Pham, and P. Ramsden, Multi-norms and the injectivity of  $L_p(G)$ , J. London Math. Soc., to appear.

\*\*\*\*\*

**Tuesday, January 17, 2012 at 4:00-5:00 pm**

Davidson Lecture Hall, Claremont McKenna College

*For more information contact Asuman G. Aksoy at [aaksoy@cmc.edu](mailto:aaksoy@cmc.edu)*